Technical Notes

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A Model for the Backflow Mean Velocity Profile

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Introduction

DETAILED flow structure data obtained with a laser anemometer for the near wall backflow region of a separating turbulent boundary layer have been presented in Refs. 1-3. Downstream of fully developed separation[4] ($\tau_w = 0$ and the fraction of time that the flow moves downstream near the wall, $\gamma_{puw} = 1/2$), the mean backflow region appears to be divided into three layers: a viscous layer nearest the wall, an intermediate layer that seems to act as an overlap region between the viscous wall and outer regions, and the outer backflow region that is really part of the large-scale outer region flow. Mixing-length and eddy-viscosity models are physically meaningless in the backflow near the wall, being imaginary and negative, respectively.

It was clear from these data, which are shown in Fig. 1, that the normalized mean velocity $U/|U_N|$ was approximately a function of y/N, where U_N is the maximum streamwise backflow mean velocity and N is the distance from the wall to this maximum backflow velocity. At that time there was no obvious simple model equation that would fit the mean velocity profiles of the backflow region. Aside from the small amount of laser anemometer data of Hastings[5] and one pulsed-wire anemometer profile of Westphal,[6] which are also shown on Fig. 1, no other investigators of separated flows have obtained measurements sufficiently close to the test wall to describe this backflow region.

The purpose of this Note is to present simple model equations for the mean backflow near the wall that are consistent with experimental observations. These equations satisfy the current need for a near wall mean backflow model for use in calculation methods.[4]

Review of Physical Observations

In the near wall backflow region, Simpson et al.[2] show that the inertial and normal stress gradient terms of the streamwise Reynolds-averaged momentum equation are negligible, producing

$$\frac{\nu \partial^2 U}{\partial y^2} - \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial(-\overline{uv})}{\partial y} = 0 \qquad (1)$$

The viscous term is important for $y/N<0.2$ and is balanced by the pressure gradient term since $\partial(-\overline{uv})/\partial y$ is only significant for $y/N>0.02$. Nearest the wall where the Reynolds stress gradient can be neglected, Eq. (1) can be integrated to produce

$$\frac{U}{|U_N|} = -C\left(\frac{y}{N}\right) + \frac{P'}{2}\left(\frac{y}{N}\right)^2 \qquad (2)$$

for $y/N<0.02$, where

$$P' = \frac{N^2}{|U_N|\nu\rho}\frac{\partial P}{\partial x}$$

and C is a constant. For the Simpson et al. flow,[1,2] $P'<125$ and the pressure gradient term of Eq. (2) contributes little.

Farther from the wall ($y/N>0.02$) the Reynolds stress gradient term dominates the flow behavior. Simpson et al.[1,2] point out that mixing length and eddy viscosity models fail in the backflow and that the Reynolds shearing stress is related to the turbulence structure and not to the local mean velocity gradient. The mean velocity profiles in the backflow are a result of the time averaging of the large turbulent fluctuations and are not related to the cause of the turbulence. Turbulence energy production near the wall is negligible; turbulence energy diffusion toward the wall is responsible for the turbulence energy which is dissipated near the wall. Since no adequate turbulence model for the backflow region is available currently, similarity, dimensional, and asymptotic behavior considerations will be used to obtain a tentative backflow mean velocity profile model.

Turbulent Backflow Mean Velocity Profile

As mentioned earlier, $U/|U_N|$ appears to be a function of y/N. A U/U_τ vs yU_τ/ν law-of-the-wall velocity profile is not consistent with this correlation since both $|U_N|$ and N increase with streamwise distance, while the law-of-the-wall length scale ν/U_τ varies inversely with the velocity scale U_τ.

However, Fig. 1 shows a definite semilogarithmic region for $0.02<y/N<0.2$. The only known reasoning for this functional behavior is a Millikan-type argument. In other words, if nearest the wall the viscosity has some effect and

$$U/U_\tau = f(yU_\tau/\nu) \qquad (3)$$

and farther away from the wall

$$U/|U_N| = g(y/N) \qquad (4)$$

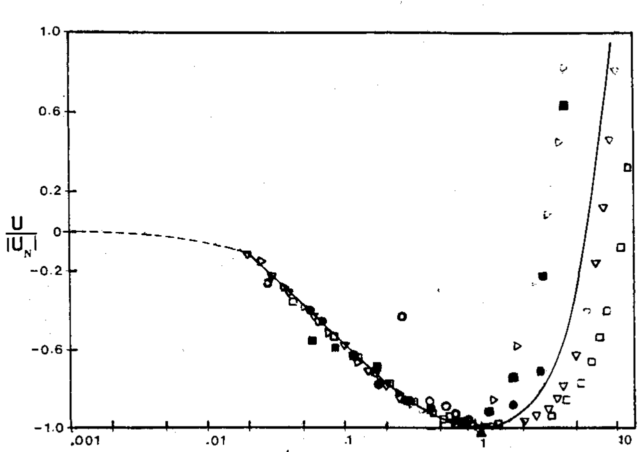

Fig. 1 Normalized backflow mean velocity profiles. Simpson et al.[1] LDV: ■, 3.53 m; ▷, 3.68 m; □, 3.97 m; and ▽, 4.34 m; Hastings[5] LDV ○; Westphal[6] pulsed-wire ●; ——, Eq. (7); and – – –, Eq. (2).

Received May 13, 1982.

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then, if an overlap region exists, one obtains

$$\frac{U_\tau^2}{\nu} f' = \frac{\partial U}{\partial y} = \frac{U_N}{N} g' \qquad (5)$$

within this region.

Since the right-hand side of this equation is independent of the kinematic viscosity ν, so must the left-hand side and $f' \sim 1/(yU_\tau/\nu)$. Thus, $g' \sim 1/(y/N)$ and one obtains

$$\frac{U}{|U_N|} = -A \ell n \left| \frac{y}{N} \right| - B \qquad (6)$$

in this overlap region. For the data shown in Fig. 1, $A = 0.3$ and $B = 1.3$.

Clearly, Eq. (6) does not describe the $U/|U_N|$ vs y/N profile for $y/N > 0.2$. If another function (y/N) is added to Eq. (4), the $U/|U_N|$ can be forced to -1 at $y = N$. A candidate function is y/N since the outer region mean velocity profile $0.4 < y/\delta < 0.8$ appears to be very linear for the far downstream detached flow locations where $\gamma_{puw} \ll 1/2$. Thus, we can write

$$\frac{U}{|U_N|} = A\left(\frac{y}{N} - \ell n \left| \frac{y}{N} \right| - 1\right) - 1 \qquad (7)$$

since this equation produces $U/|U_N| = -1$ and $\partial U/\partial y = 0$ at $y/N = 1$ and $B = 1 + A$. The term Ay/N contributes little to $U/|U_N|$ near the wall. The use of only one empirical constant A is a particularly striking feature of Eq. (7).

Figure 1 shows that Eq. (7) closely describes the mean velocity profile for $0.02 < y/N < 1.0$ with $A = 0.3$. It should be noted that experimental values of N are $\pm 15\%$ uncertain. For $y/N > 1$, the data at 3.53 m ($\gamma_{puw} = 0.33$) and 3.68 m ($\gamma_{puw} = 0.16$) and Westphal's data ($\gamma_{puw} = 0.02$) for a backward facing step are higher than Eq. (7). For 3.97 m ($\gamma_{puw} = 0.1$) and 4.34 m ($\gamma_{puw} = 0.06$) the data fall below Eq. (7).

For $y/N < 0.02$, Eq. (2) can be used to describe the velocity profile of the viscous wall layer and to estimate the mean surface shearing stress when C is determined at $y/N = 0.02$. For example, at 4.34 m, $P' = 120$ and $C_f/2 = 1 \times 10^{-4}$. This skin friction coefficient is about an order of magnitude smaller than upstream of detachment. For the 4.34 m data, $y/N = 0.02$ corresponds to $yU_\tau/\nu = 2.6$. This is a reasonable number in view of the viscous sublayer thickness for attached boundary layers, $yU_\tau/\nu \approx 5$.

Conclusions

Equation (7) can be used to describe the velocity profile of the middle region of the mean backflow, $0.02 < y/N < 1.0$, when $\gamma_{pu} < 1/2$ near the wall. Laser anemometer and pulsed-wire anemometer data both support this mean velocity profile description. Nearer the wall, the viscous layer can be described by Eq. (2), although no experimental data in this region have been obtained. Farther away from the wall, $y/N > 1.0$, Eq. (7) does not describe the mean velocity profile well since this outer backflow region is influenced strongly by the large-scale outer region flow.

Equations (2) and (7) can be used as near wall backflow functions for separated flow calculation methods when $\gamma_{pu} < 1/2$. Since these equations describe the portion of the mean backflow where $\partial U/\partial y \leq 0$, they eliminate the need for a calculation method to model the near wall flow where $\partial(-\overline{uv})/\partial y$ and $\partial U/\partial y$ are of opposite signs.

Acknowledgments

This work was done in conjunction with work supported by the Experimental Fluid Dynamics Branch of NASA Ames Research Center and work supported by the Office of Naval Research.

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Upstream Influence in Sharp Fin-Induced Shock Wave Turbulent Boundary-Layer Interaction

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Introduction

THREE-dimensional shock wave turbulent boundary-layer interactions are complex phenomena which occur in a wide variety of practical internal and external aerodynamic problems. Common examples are the sidewall interactions in engine inlets, and protuberance and fin-induced flowfields. Up until the last few years, most studies have been experimental. The recent rapid development of computational fluid dynamics has now made it possible to simulate numerically certain three-dimensional interactive flowfields, such as the skewed shock wave interaction.[1] Overall, good comparisons of numerical predictions with experiment have been obtained.

This flowfield has been the subject of many investigations, including the current study. The interaction is generated easily using the model configuration sketched in Fig. 1. It consists of an unswept, semi-infinite sharp fin at angle of attack α mounted normal to a flat surface on which the incoming boundary layer develops. A swept interaction is generated whose scale increases in the spanwise direction. Although much has been learned about this flowfield from such detailed experimental studies as those of Oskam et al.[2] and Peake,[3] many critical questions remain unanswered. One of the most fundamental is that of which parameters control the interaction scale and spanwise growth.

In an attempt to provide a partial answer to this question, measurements have been made of how a primary interaction length scale, namely, upstream influence, develops spanwise in this flowfield. Fin models have been tested at various α in

Received July 24, 1981; revision received May 6, 1982.

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